Rosen 1, Goertzel 0:
Comments on the appendix “Goertzel versus Rosen”

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1. Introit

In this note I shall point out some errors in Ben Goertzel’s essay “Goertzel versus Rosen: Contrasting views on the autopoietic nature of life and mind” [1]. Goertzel claimed that one could, using division algebras, construct computable systems fulfilling Rosen’s mathematical definition of a cell (i.e. an (M,R)-system). If such constructions were valid, they would of course provide counterexamples to one of Rosen’s central theorems, that “any material realization of the (M,R)-system must have noncomputable models” (and all the variations and corollaries). I shall show that Goertzel’s constructions place his mappings at the wrong levels in the categorical hierarchy, and therefore what he constructed were, in fact, *not* (M,R)-systems.

2. Notations

More for the notations than for the details, I need to give a brief description of Rosen’s (M,R)-system here. I assume that if you are not familiar with it, you would not be reading this note in the first place. I must, however, emphasize that the two most important references on (M,R)-systems are [2] and [3]. One should go back to these two papers for the fine nuances.

The simplest (M,R)-system may be represented by the diagram

$$A \xrightarrow{f} B \xrightarrow{\Phi} H(A,B) \tag{1}$$

Note the adjective *simplest* here. This form (1) was what Rosen used in almost all of his subsequent discussions on (M,R)-systems. But remember that a general (M,R)-system is actually a network of metabolism and repair components. It is true that form (1) captures the essence of all (M,R)-systems, and indeed it is possible in principle to reduce every abstract (M,R)-system to this simple form by making A, B, and f sufficiently complex. One must, nevertheless, not lose sight of the network aspect of (M,R)-systems. See [3], in particular.

There are three ways to show the function (or *morphism* in categorical language) f with its domain A and codomain B:

$$A \xrightarrow{f} B \tag{2}$$

$$f: A \to B \tag{3}$$

$$f \in H(A,B) \tag{4}$$

I use them interchangeably.

In form (1), the function f represents metabolism (enzyme):

Metabolism	$f: A \rightarrow B$	$f \in H(A,B)$

(5)

The function Φ represents repair (gene). It repairs the metabolism function, in the sense that since its codomain is $H(A,B)$, it may be considered as a function that creates new copies of f.

Repair	$\Phi: B \rightarrow H(A,B)$	$\Phi \in H[B, H(A,B)]$

(6)

What if the repair components themselves need repairing? New functions representing replication (i.e. that serve to replicate the repair, or genetic, components) may be defined. A replication map must have as its codomain the hom set $H[B, H(A,B)]$ to which repair functions Φ belong, whence it must be of the form

$$\beta: Y \rightarrow H[B, H(A,B)] \tag{7}$$

for some set Y. For the convenience of iterative combination discussed immediately below, we choose $Y = H(A,B)$, so

$$\beta: H(A,B) \rightarrow H[B, H(A,B)]\,. \tag{8}$$

Replication	$\beta: H(A,B) \rightarrow H[B, H(A,B)]$	$\beta \in H\big[H(A,B), H[B, H(A,B)]\big]$

(9)

The morphism (8) may be combined with the second morphism in (1) to give a new (M,R)-system from the old one; viz.

$$B \xrightarrow{\ \Phi\ } H(A,B) \xrightarrow{\ \beta\ } H[B, H(A,B)] \tag{10}$$

which has the property that the "metabolic" part of system (10) is the "repair" part of system (1), and the "repair" part of system (10) is the "replication" part of system (1) (i.e. form (8)). Indeed, one may sequentially extend this formalism ad infinitum, the next system being

$$H(A,B) \longrightarrow H[B, H(A,B)] \longrightarrow H\big[H(A,B), H[B, H(A,B)]\big]\,. \tag{11}$$

If this were all there is to it with (M,R)-systems, it would have been pretty pointless. The magic of an (M,R)-system is that the replication function (8) is already entailed in the original form (1). On the basis of what are already present in (1), "under stringent but not prohibitively strong conditions, such replication essentially comes along for free." So no infinite sequence of arrows here; arrow diagram (1) alone suffices.

Note no "inverse" has yet been mentioned up to this point. We have used a generic symbol β for the replication map. There are many ways to construct β from nothing else but what are already in the arrow diagram (1). Rosen has always used the simplest way, that of an inverse evaluation map. But it is most certainly not the only way. The most important aspect of a replication map is that it needs to produce repair functions Φ, which belong the hom set $H[B, H(A, B)]$. Therefore the codomain of a replication map β must be $H[B, H(A, B)]$. The fact that Rosen's β turns out to be an inverse evaluation map is entirely incidental.

Here is how one constructs Rosen's β. An element $b \in B$ defines an "evaluation map"

$$\hat{b} \in H\big[H[B, H(A, B)], H(A, B)\big] \tag{12}$$

by

$$\hat{b}(\Phi) = \Phi(b) . \tag{13}$$

The function $\hat{b}$ is invertible if it is monomorphic; viz. $\forall \Phi_1, \Phi_2 \in H[B, H(A, B)]$,

$$\hat{b}(\Phi_1) = \hat{b}(\Phi_2) \Rightarrow \Phi_1 = \Phi_2 ; \tag{14}$$

i.e.

$$\Phi_1(b) = \Phi_2(b) \Rightarrow \Phi_1 = \Phi_2 . \tag{15}$$

This implication (15) is a condition on the repair maps $\Phi \in H[B, H(A, B)]$: if two repair maps agree at b, then they must agree everywhere. In other words, a repair map Φ is uniquely determined by its one value $\Phi(b)$. These are essentially the "stringent but not prohibitively strong conditions" required to make the inverse evaluation map a replication map with nothing but the ingredients of arrow diagram (1).

Note the inverse evaluation map $\hat{b}^{-1}$ maps thus:

$$\hat{b}^{-1} : H(A, B) \to H[B, H(A, B)] , \tag{16}$$

$$\hat{b}^{-1}\big(\Phi(b)\big) = \Phi . \tag{17}$$

It takes one function value $\Phi(b) \in H(A, B)$ to the whole function $\Phi \in H[B, H(A, B)]$: this is the sense in which it "replicates". But the stringent condition requiring that a repair map Φ to be uniquely determined by its one value $\Phi(b)$ neatly overcomes this $\Phi = \Phi(b)$ identification problem!

3. Goertzel's construction

Goertzel only stated "that B is the set of nodes in Webmind", without defining what the set A is. And "$H(A,B)$ is then the set of mappings from nodes into nodes." We may either assume $A = B$, or that they are two different sets of nodes: it makes no difference in the subsequent discussions. Thus Goertzel's version of Rosen's metabolism map (5) is

node mapping	$f : A \to B$	$f \in H(A,B)$

(18)

(By another name, node mapping = node transformer.)

The big mistake in Goertzel's construction is his next step, when he said "we can regard a node as an operator on node mappings, by the logic node^(node mapping)=node mapping(node)". In other words, for a node $n \in A$ and a node mapping $f \in H(A,B)$,

$$\hat{n}(f) = f(n) . \tag{19}$$

"The node's action on a mapping is to tell you what the mapping maps the node into"; he calls such a mapping $\hat{n}$, which takes node mappings in to nodes, "node transformer projector". But note the domain and codomain of $\hat{n}$:

node transformer projector	$\hat{n} : H(A,B) \to B$	$\hat{n} \in H[H(A,B),B]$

(20)

Compare (20) with Rosen's evaluation map (12) above. See the discrepancy in the domains and codomains?

Goertzel then continued, and defined a "node awakener" as the inverse of a "node transformer projector" (hence taking nodes into node mappings):

node awakener	$\hat{n}^{-1} : B \to H(A,B)$	$\hat{n}^{-1} \in H[B, H(A,B)]$

(21)

Compare (21) with Rosen's inverse evaluation map (16) above. Again observe the discrepancy in the domains and codomains.

If we were to assemble Goertzel's maps into something that resembles an (M,R)-system, it would look like this:

$$A \xrightarrow{f} B \xrightarrow{\hat{n}^{-1}} H(A,B) . \tag{22}$$

So it is apparent that Goertzel went through this whole inverse evaluation map exercise for the "repair" step, rather than for the "replication" step. Using his terminology, instead of his "node^(node mapping)=node mapping(node)" construction, what he should have used is "node^(node awakener)=node awakener(node)". Goertzel's "inverse evaluation map" is a node awakener, which takes nodes into node mappings. Rosen's "inverse evaluation map" for (M,R)-systems takes node mappings into node awakeners. Goertzel missed it by one level in the categorical hierarchy.

In an (M,R)-system, the repair functions $\Phi \in H[B, H(A, B)]$ are primary ingredients in form (1). It is their own closed entailment that makes (M,R)-systems special. Putting it another way, in (M,R)-systems, morphisms with codomain $H[B, H(A, B)]$ come along for free from (1). In Goertzel's system (22), all he has constructed is something with $H(A, B)$ as the codomain.

4. Division algebra

Since the error in Goertzel's construction occurs because of categorical hierarchy, we do not even have to go to the model of his construction in terms of division algebras. But the latter in fact misses yet another level in the hierarchy!

Definition A *division algebra* (also called a division ring) is a ring in which every nonzero element has a multiplicative inverse.

I will not define what a ring is here. Just think of the set of real numbers with its 0 and 1, with addition and multiplication, and all the nice algebraic properties that go along with them. The set of real numbers $\mathbb{R}$ is a division algebra. Multiplication is not required to be commutative; but if it is, the division algebra is called a field. So $\mathbb{R}$ is a field.

Goertzel's statement that a "vector multiplication operator" is invertible "under reasonable algebraic conditions only for k=1,2,4,8" is a paraphrase of the

Theorem The only finite-dimensional real division algebras occur for dimensions k =1, 2, 4, and 8, corresponding to real numbers, complex numbers, quaternions, and Cayley numbers, respectively.

Real numbers and complex numbers are fields. Multiplication for quaternions is not commutative. Multiplication for Cayley numbers (also called octonions) is not even associative, although it is a requirement in the definition of division algebra. So we get around it by saying octonions are a *nonassociative* division algebra.

So what do division algebras have to do with (M,R)-systems? The two concepts of "inverse" appearing in each are not even the same thing! In division algebras we worry about *multiplicative* inverse, while in (M,R)-systems it is *functional* inverse. The trick, is to make them coincide.

Let $(B,+,*)$ be a division algebra, and choose an element $b \in B$. Then the "multiplication by a fixed element" function may be defined as

$$m_b : B \to B \qquad (\text{i.e. } m_b \in H(B,B)\ ) \tag{23}$$

by $\forall x \in B$,

$$m_b(x) = b * x\ . \tag{24}$$

(Note that the hom set $H(B, B)$ is in the category of division algebras here, so has all the properties that go with this.) The inverse function

$$m_b^{-1} : B \to B \quad \text{(hence also } m_b^{-1} \in H(B, B) \text{)} \tag{25}$$

exists if the element b itself has a multiplicative inverse b^{-1}. In this case we can equate m_b^{-1} to "multiplication by the fixed element b^{-1}" (i.e. $m_b^{-1} = m_{b^{-1}}$): then $\forall x \in B$,

$$m_b^{-1}(m_b(x)) = m_b^{-1}(b * x) = b^{-1} * (b * x) = (b^{-1} * b) * x = 1 * x = x \ . \tag{26}$$

Note that in the $b^{-1} * (b * x) = (b^{-1} * b) * x$ part of (26), we have used the associativity of multiplication. So if we are only concerned with finite-dimensional real division algebras, this rules out k =8 the Cayley numbers. We are left with k =1, 2, and 4.

The fatal error in this division algebra model, though, is that in the "multiplication by a fixed element" function, we necessarily have to have the same domain and codomain, both being the division algebra B; whence $m_b \in H(B, B)$ and $m_b^{-1} \in H(B, B)$ (as in (23) and (25)).

multiplicative inverse	$m_b^{-1} : B \to B$	$m_b^{-1} \in H(B, B)$	(27)

So we see that "multiplicative inverse" as a map actually is one step even lower in the categorical hierarchy than Goertzel's "inverse evaluation map" in the previous section. It cannot even model Goertzel's node awakener (21). This map (27) is at the level of metabolism, definitely not a candidate for replication.

5. References

[1] Ben Goertzel (2002) Appendix 2 (pp.289–295) in *Creating Internet Intelligence*. Kluwer Academic/Plenum Publishers, New York.

[2] Robert Rosen (1971) Some realizations of (M,R)-systems and their interpretation. *Bull. Math. Biophys*. 33, 303–319.

[3] Robert Rosen (1972) "Some relational cell models: the metabolism-repair systems". Chapter 4 (pp.217–253) of *Foundations of Mathematical Biology*, Vol. 2. Academic Press, New York.